

Microwave pinning modes near Landau filling $\nu = 1$ in two-dimensional electron systems with alloy disorder

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Abstract

We report measurements of microwave spectra of two-dimensional electron systems hosted in dilute Al alloy, $\text{Al}_x\text{Ga}_{1-x}\text{As}$, for a range of Landau level fillings, ν , around 1. For $\nu > 0.8$ or $\nu < 1.2$, the samples exhibit a microwave resonance whose frequency decreases as ν moves away from 1. A resonance with this behavior is the signature of solids of quasiparticles or -holes in the partially occupied Landau level, which was previously seen in ultralow disorder samples. For $\nu < 0.8$ down to as low as $\nu = 0.54$, a resonance in the spectra is still present in the Al alloy-disordered samples, though it is partially or completely suppressed at $\nu = 3/5$ and $1/2$, and is strongly damped over much of this ν range. The resonance also shows a striking enhancement in peak frequency for ν just below $3/4$. We discuss possible explanations of the resonance behavior for $\nu < 0.8$ in terms of the composite fermion picture.

In two-dimensional electron systems (2DES) in high magnetic field, B , Wigner solids^{1–20} pinned by residual disorder occur for sufficiently small Landau level filling factor, ν . At the low ν termination of the fractional quantum Hall effect (FQHE) series, such a solid can appear in dc transport as an insulating phase. By means of microwave spectroscopy like that employed to obtain the data in this paper, Wigner solids of dilute quasiparticles or quasiholes in the presence of one or more filled Landau levels have also been found^{21–23}, in extremely high mobility, low disorder 2DES. This type of solid has been named an integer quantum Hall effect Wigner solid (IQHEWS) since the vanishing diagonal conductivity in the wings of the integer quantum Hall effect (IQHE) diagonal conductivity minimum is due to the pinning of the solid. An analogous solid composed of quasiparticles and -holes of the $1/3$ fractional quantum Hall liquid has also been reported²⁴.

Microwave spectroscopy is known to directly measure pinned solids in high magnetic field, since these solids exhibit resonances in their microwave spectra^{10,13,18,21–25}. The resonances are understood as pinning modes^{13–18,26}, in which pieces of the solid oscillate within the disorder potential. The frequency, f_{pk} , of the pinning mode is determined by the disorder in the sample and by the stiffness of the solid. f_{pk} always increases when the disorder strength is increased, but in weak pinning theory^{26–29} when the solid is made stiffer, f_{pk} decreases. This behavior, verified experimentally^{17,18,21}, occurs because for a stiffer crystal the carrier is less closely associated with disorder potential features. In the case of the IQHEWS^{21,22}, near $\nu = 1$ the quasiparticle or -hole density is $n^* = |\nu^*|n/\nu$ where $\nu^* = \nu - 1$, and n is the carrier density of the 2DES. Hence the generic behavior of the IQHEWS is that f_{pk} is largest near the integer ν , and the resonance initially increases in strength as one moves away from integer ν and $|\nu^*|$ increases. At still larger $|\nu^*|$, above 0.15 in low disorder samples²¹ near $\nu = 1$, the resonance fades away as the carriers become dense enough for the solid to melt.

Earlier reports of IQHEWS have been on quantum well samples of the highest quality. These include²² a 50 nm well, with mobility $\mu \approx 15 \times 10^6$ cm²/V-s, and a 30 nm well with $\mu \approx 27 \times 10^6$ cm²/V-s. Here we report IQHEWS, identified from microwave spectroscopy, in samples with far lower mobility, 1.4 and 2.3×10^6 cm²/V-s. The samples used in this work contain the 2DES in dilute Al alloy, $\text{Al}_x\text{Ga}_{1-x}\text{As}$, so the isoelectronic Al impurities provide an additional component of the overall disorder. The length scales of this scattering potential are much less than the magnetic length, l_B , under all conditions in this paper.

This paper shows that even with the Al alloy disorder, for $|\nu^*| < 0.2$ the resonances

mainly follow the behavior expected of an IQHEWS resonance, with f_{pk} increasing as $\nu = 1$ is approached. f_{pk} vs ν , also shows inflections likely due to skyrmion crystallization²². Furthermore, more detailed study of the pinning resonance also shows that relative to samples without Al, the resonance range is greatly extended on the hole side ($\nu < 1$) down to $\nu \sim 0.6$. In this extended range, the behavior of the resonances is more complex than was reported²¹ in Al-free samples, and the interpretation may require phase transitions within the solid. As ν decreases below 0.8, the trend of decreasing f_{pk} reverses, and f_{pk} vs ν develops a strongly disorder-dependent maximum for ν just below $3/4$. The resonance also is suppressed or partly suppressed at $\nu = 2/3$ and $3/5$. Particularly away from $\nu = 1$ and from the maximum in f_{pk} vs ν , the resonance is highly damped with a rapid rise at low frequency, f , in real diagonal conductivity, $\text{Re}(\sigma_{xx})(f)$. We observed such a highly damped resonance in the ranges of ν between $2/3$ and $3/5$ reported³⁰ to exhibit insulating behavior in the partially filled Landau level in dc transport

We present data from two samples with $\text{Al}_x\text{Ga}_{1-x}\text{As}$ channels. Samples from the same wafers were used in earlier work in dc transport^{30–34}. Characterization³¹ of the samples at $B = 0$ showed that the spatial distribution of the Al is random. Both samples are single heterojunctions with the 2DES approximately 180 nm below the top surface of the sample. One sample has $x = 0.33\%$, density $n \approx 2.1 \times 10^{11} \text{ cm}^{-2}$ and as cooled mobility μ about $2.3 \times 10^6 \text{ cm}^2/\text{V-s}$; the other sample has $x = 0.85\%$, $n \approx 2.5 \times 10^{11} \text{ cm}^{-2}$ and $\mu \approx 1.4 \times 10^6 \text{ cm}^2/\text{V-s}$. A previous paper³⁵ showed that samples from this series exhibit pinning mode resonances at the low ν termination of the FQHE series, which for these disordered samples occurred for ν below that of the $1/3$ FQHE. The presence of disorder is known to raise the ν at which the FQHE series is terminated in an insulator. At least for samples grown in the same series, for which the component of disorder not due to the Al was the same, additional Al raised f_{pk} , as one would expect for a pinning mode.

Microwave spectra of the complex diagonal conductivity σ_{xx} , were obtained from the propagation characteristics of a transmission line¹⁰ that is capacitively coupled to the 2DES. The transmission line we used is shown schematically in Fig. 1a. The lines are of the coplanar waveguide (CPW) type^{17,18,21–25,36}, with a center conductor separated from two broad ground planes by slots of width $W = 30 \mu\text{m}$, and are patterned directly onto the top of the sample. We present complex diagonal conductivity calculated from the model of ref. 36 for the CPW coupled to the 2DES. The results are within about 25% of those from the high frequency low

loss approximation, $\sigma_{xx} \approx -W|\ln(s)|/Z_0d$, where s is a complex transmission coefficient, $Z_0 = 50 \Omega$ is the characteristic impedance calculated from the transmission line geometry for $\sigma_{xx} = 0$, and d is the length of the coplanar waveguide ($d = 28$ mm). s is normalized to the transmission at $\nu = 1$, at which σ_{xx} vanishes due to the IQHE. For all measurements, the 2DES temperature was approximately 60 mK, as read by nearby resistance thermometers. Slight broadening of the resonance could be discerned on increasing the temperature above that value. We verify that the data were taken in the low microwave power limit, in which further decrease of power does not affect the measured spectrum.

Figure 1b shows $\text{Re}(\sigma_{xx})$ vs B for the two samples, both taken at 100 MHz. $\text{Re}(\sigma_{xx})$ for the $x = 0.33\%$ sample shows a broad minimum at the $\nu = 1$ IQHE, and FQHE minima at $3/5, 2/3, 4/3, 7/5, 8/5$ and $5/3$. The curve is composed of points read off from spectra taken at discrete B , and the density of points gives the traces an artifactual jagged appearance near $\nu = 3/5$ and $2/3$. The width of the IQHE minimum is about 0.3 in ν , measured from halfway up the rises on either side. The trace from the $x = 0.85\%$ sample shows a much wider IQHE around $\nu = 1$, with ν -width measured the same way of 0.56. The FQHE at $2/3$ or $3/5$ is not apparent, though there are minima near $4/3$ and $5/3$. For both traces, the $\nu = 2$ IQHE minimum is well-developed but appears at $\text{Re}(\sigma_{xx})$ slightly elevated from that at $\nu = 1$; we ascribe this to weak, B -dependent parallel conduction in the wafers.

Figure 2a is an image plot of spectra, $\text{Re}(\sigma_{xx})$ vs f , for ν between 0.6 and 1.4 for the $x = 0.33\%$ sample. The $x = 0.33\%$ trace in Fig. 1b is a 0.1 GHz cut of this image. There are clear resonances on each side on $\nu = 1$, which decrease in f_{pk} as ν moves farther from 1. As ν goes above 1.2, the resonance fades into a flat spectrum. In contrast, as ν goes below 0.8, $\text{Re}(\sigma_{xx})$ vs f continue to exhibit maxima.

$\text{Re}(\sigma_{xx})$ spectra for all $\nu \leq 1$ and $x = 0.33\%$ are shown in Fig. 3a. For ν between 0.8 and $2/3$, $\text{Re}(\sigma_{xx})$ shows a strongly asymmetric peak, with rapid rise at low f and a high f tail, whose extent is ν dependent. At $2/3$ the spectrum becomes flat. The asymmetric peak reappears for $\nu < 2/3$, but weakens for the lowest ν trace at 0.61.

The color image in Fig. 2b gives an overview of the $\text{Re}(\sigma_{xx})$ spectra for the $x = 0.85\%$ sample, and the spectra for $\nu \leq 1$ are plotted as curves in Fig. 4a. Between $\nu = 1.05$ and 1.2, and also between between $\nu = 0.94$ and 0.8 there is a resonance whose f_{pk} increases as $\nu = 1$ is approached. As ν goes below 0.8, the resonance moves to higher f_{pk} , with a maximum f_{pk} around $\nu = 0.74$. As $2/3$ is approached from higher ν the resonance decreases in frequency

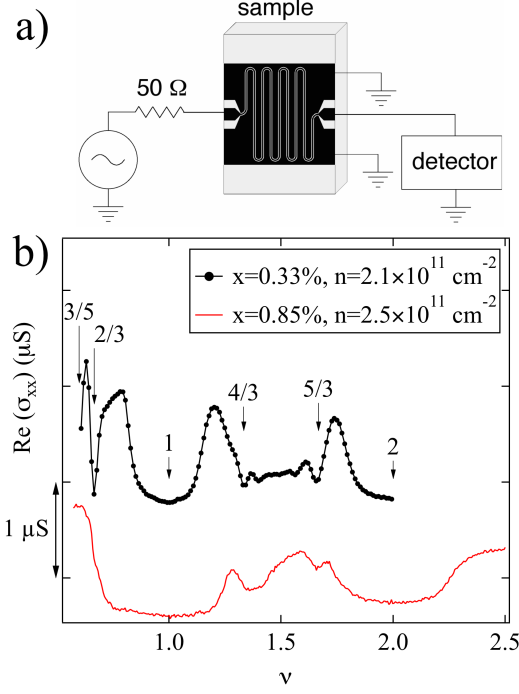


FIG. 1: a) Schematic representation of the microwave circuit used in our measurement, not to scale. Black areas represent metal films on the sample surface. b) Real part of diagonal conductivity $\text{Re}(\sigma_{xx})$, vs Landau filling factor ν for the two samples at 0.1 GHz. The upper, ($x = 0.33\%$) curve was interpolated from spectra, and has a lower density of data points, which are shown as dots.

and gradually becomes asymmetric. A resonance is present at $2/3$ and $3/5$, which lie within the low- f IQHE plateau, but these rational fractional ν are easily distinguished in Fig. 2b by their reduced overall $\text{Re}(\sigma_{xx})$.

To characterize the spectra it is useful to perform fits to extract resonance parameters, particularly when the resonances are asymmetric and broad. Figures 3b and 4b show $\text{Im}(\sigma_{xx})$ vs f at several fillings as marked. The $\text{Im}(\sigma_{xx})$ spectra can be fit to the imaginary part of the response of a Drude-Lorentz harmonic oscillator³⁷, $\sigma_{xx}(f) = \{\sigma_0 f \Delta f / [f \Delta f + i(f^2 - f_{\text{pk}}^2)]\}$, a form commonly used³⁷ for dielectrics, where the parameters σ_0 and Δf are the amplitude and line width. The corresponding real part, with the same fit parameters, fit the $\text{Re}(\sigma_{xx})$ spectra to within frequency-independent additive constants. The fit, common for lossy dielectrics³⁷ was chosen empirically, and is not readily connected to theoretically derived forms²⁷ of pinning modes for weak disorder in high B , possibly since these existing theories are specific to the weak disorder limit.

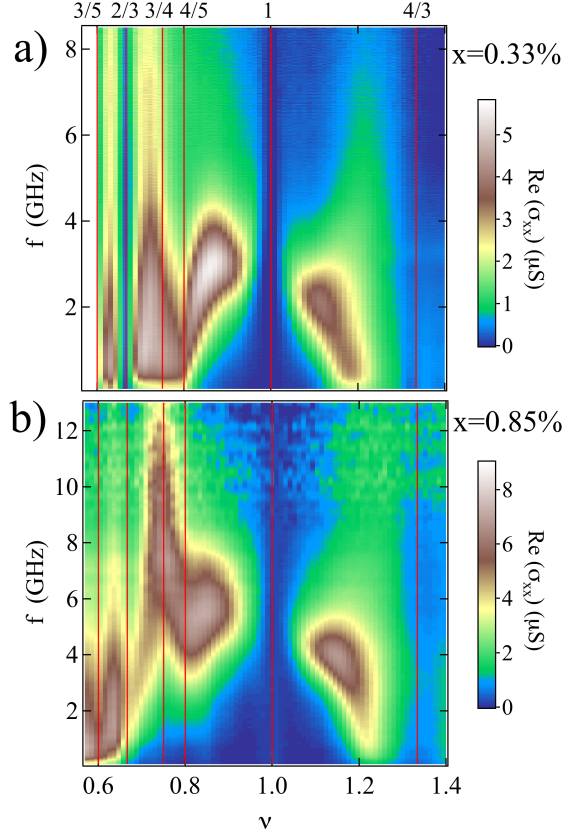


FIG. 2: Color-scale plots showing $\text{Re}(\sigma_{xx})$ in f - ν plane a) $x = 0.33\%$ b) $x = 0.85\%$ Red vertical lines mark Landau filling factors $3/5, 2/3, 3/4, 1$ and $4/3$.

Figure 5 shows f_{pk} and the linewidth, Δf , vs ν and ν^* for both the samples. Both f_{pk} and Δf come from fits of $\text{Im}(\sigma_{xx})$ vs f , like those shown in Fig. 3 and Fig. 4. Figure 5a also shows f_{pk} vs ν from ref. 22, on an ultralow disorder quantum well sample, with $x = 0$, $n = 2.7 \times 10^{11} \text{ cm}^{-2}$ and $\mu \approx 27 \times 10^6 \text{ m}^2/\text{V-s}$. The $x = 0.33$ and 0.85% f_{pk} traces are decreasing as ν moves away from 1, out to $|\nu^*| \approx 0.2$. f_{pk} vs ν for the $x = 0$ ultralow disorder sample likewise decreases on moving away from 1, though the range of ν for that data is less. Inflections are marked with arrows for the f_{pk} vs ν traces for $x = 0.33\%$ and for $x = 0.85\%$, but are also visible in the $x = 0$ data, for slightly smaller $|\nu^*|$. Inflections are marked by down arrows for the $x = 0.33\%$ f_{pk} vs ν data in Fig. 5a, at $\nu = 0.91$ and 1.08 . Inflections are also visible in the $x = 0$ f_{pk} vs ν , for slightly smaller $|\nu^*|$. A similar but much weaker feature in $\nu \approx 0.88$ is marked in Fig. 5b as well.

The main point of this paper is that the behavior of the resonance for $|\nu^*| < 0.2$, with monotonic decrease of f_{pk} as ν^* is evidence for an IQHEWS, much like that found in ultralow

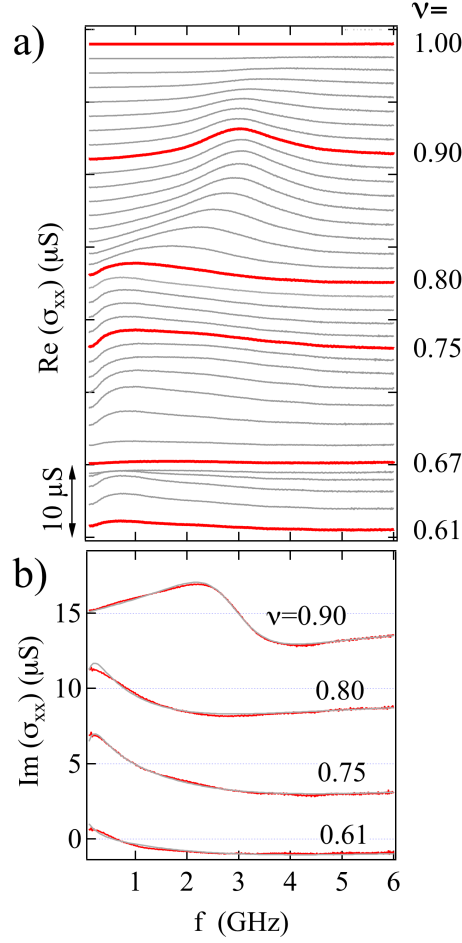


FIG. 3: Spectra for the sample with $x = 0.33\%$ for different Landau level fillings ν . a) $\text{Re}(\sigma_{xx})$ vs frequency, f . Successive spectra were taken at ν -intervals of 0.0107, and are offset vertically proportionally to ν . Landau fillings of certain spectra (red) are marked at right. b) $\text{Im}(\sigma_{xx})$ vs f for several ν . Successive spectra are offset by $5 \mu\text{S}$. Fits are light lines, data are darker (red) lines.

disorder samples. The decrease in f_{pk} is due to the increase in n^* , which results in a stiffer solid that avoids features of the disorder which increase pinning. f_{pk} is larger than the ultralow disorder sample, as expected for larger disorder, but the disorder, possibly because of its homogenous distribution and short range, still allows IQHEWS formation.

The inflections in the f_{pk} vs ν traces of Fig. 5 are also similar to effects in the $x = 0$ ultralow disorder sample. The inflections can be regarded as due to a sudden decrease in the pinning as ν approaches 1, superposed on the pinning increase engendered by the reduction in quasiparticle density. Based on ref. 22 we interpret this as due to formation of

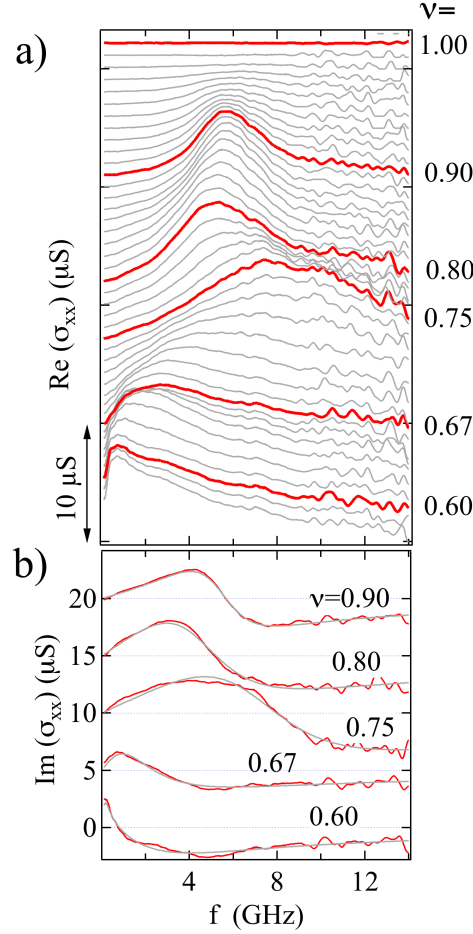


FIG. 4: a) Spectra for the sample with $x = 0.85\%$ for different Landau level fillings ν . a) $\text{Re}(\sigma_{xx})$ vs frequency, f . Successive spectra were taken at ν -intervals of 0.0111, and are offset vertically proportionally to ν . Landau fillings of certain spectra (red on line) are marked at right. b) $\text{Im}(\sigma_{xx})$ vs f for several ν . Successive spectra are offset by $5 \mu\text{S}$. Fits are light lines, data are darker (red on line) lines.

skyrmions^{38–40} containing multiple spins. Crystals^{41,42} of such skyrmions can form near $\nu = 1$ state when 1) ν sufficiently close to 1 and 2) when Zeeman energy is sufficiently small relative to Coulomb energy, that is, for sufficiently small $\tilde{g} = g\mu_B B_T / (e^2 / 4\pi\epsilon_0 l_B)$, where B is total magnetic field. For the present samples $\tilde{g} = 0.017, 0.019$ respectively for the $x = 0.33, 0.85\%$, using $|g| = 0.44$. The sample that produced the $x = 0$ trace of Fig. 5a had $\tilde{g} = 0.019$ in perpendicular magnetic field; the inflections in f_{pk} vs ν of this sample were shown²² to disappear on applying in-plane magnetic field to increase \tilde{g} . In-plane field experiments on

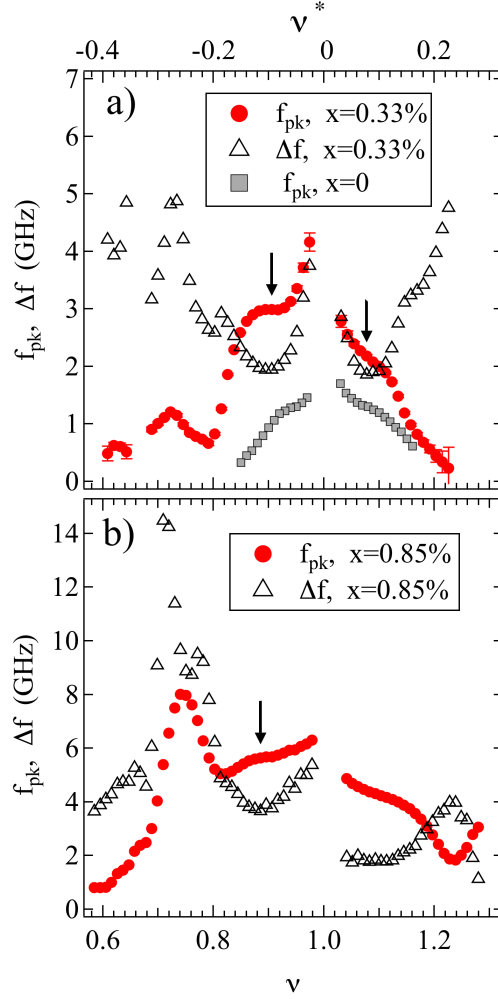


FIG. 5: a) Resonance parameters peak frequency, (f_{pk}) and linewidth, (Δf) vs Landau level filling ν . $\nu^* = \nu - 1$ is shown on the top axis. for a) $x = 0.33\%$, $x = 0$ data are f_{pk} from ref. onlinecitehanskyrme, for a 30 nm quantum well with mobility $27 \times 10^6 \text{ m}^2/\text{V-s}$. b) $x = 0.85\%$. Arrows mark the inflections discussed in the text.

the alloy-disordered samples would verify the skyrmion based interpretation. The disorder at the $x = 0.33\%$ level actually appears to make the skyrmion effect *more prominent* than in the $x = 0$ ultralow disorder sample. A skyrmion effect in the Al disordered sample also indicates a remarkable robustness of this interaction-driven phenomenon in the presence of homogenous short-range disorder.

As ν goes below 0.8 (and also above 1.2 in a small range for $x = 0.85\%$), the decrease of f_{pk} with $|\nu^*|$ reverses, and the interpretation of the resonance is less clear. For $\nu < 0.8$,

f_{pk} increases with decreasing ν and develops a peak around 0.72 for $x = 0.33\%$, and a much more pronounced peak around $\nu = 0.74$ for $x = 0.85\%$. The behavior seen for $\nu < 0.8$ is clearly enabled by the disorder, since the disorder extends the resonant range into that region. Disorder stabilizes a solid vs an FQHE liquid since the solid optimizes its energy in the disorder potential, gaining its pinning energy⁴³, and the transition to insulator at the low ν termination of the FQHE series occurs around $\nu = 1/3$ in Al alloy-disordered samples^{30,35}.

Except for a small range near $\nu = 1.2$ for $x = 0.85\%$, the overdamped ($\Delta f > f_{\text{pk}}$ on Fig. 5) regions have $|\nu^*| > 0.2$. The most strongly overdamped resonance spectra are for ν well below 1, for $\nu < 0.8$ for $x = 0.33\%$ and $\nu < 2/3$ for $x = 0.85\%$. Their spectra are characterized by a sharp rise in $\text{Re}(\sigma_{xx})$ at low f . In the ν regions where these spectra occur, the brown color in Fig. 2 comes to within 0.1 GHz of the lower axis. The spectra are qualitatively different from the better-developed resonances seen closer to integer ν , or in the low ν insulator, both in the present samples³⁵ and in low-disorder Al-free quantum wells^{21,22}.

Earlier dc transport measurements³⁰ particularly on a sample from the same $x = 0.85\%$ wafer showed a reentrant integer quantum Hall effect (RIQHE), between the $2/3$ and $3/5$ FQHEs. The RIQHE has vanishing diagonal resistance and Hall resistance quantized to h/e^2 , and indicates that quasiholes of the partially occupied Landau level are insulating. In the RIQHE ν range of ref. 30 we find resonances of the strongly overdamped type in both samples. Possibly because of sample processing, variation within the wafer, or cooldown procedure, we do not find a $2/3$ FQHE in the present $x = 0.85\%$ sample. There is a $2/3$ FQHE in the present $x = 0.33\%$ sample, so in that sample the insulator between $2/3$ and $3/5$ is reentrant.

While the interpretation of the resonance for $|\nu^*| > 0.2$ is unclear, we present different possibilities, involving the composite fermion (CF) picture⁴⁴. At rational fractional ν of FQHE states the resonance can be suppressed, hence it is reasonable to assume that the CF picture, which has explained many aspects of the FQHE, will be of value even in the presence of the disorder potential. There has been recent detailed theoretical treatment^{6,7} of composite fermion (CF) Wigner crystals in disorder-free systems. A CF Wigner crystal, denoted $^{2p}\text{CFWC}$, is composed of CFs with $2p$ flux bound to an electron. The theory, presented in ref. 7 for $\nu < 4/17$, predicts a series of transitions between different $^{2p}\text{CFWC}$ with $2p$ increasing as ν decreases, in particular a change from a $^2\text{CFWC}$ to $^4\text{CFWC}$ as ν

decreases from $1/5$ to $1/6$. Since the disorder extends ν^* for the resonance well beyond where solid in a low-disorder system is expected, the theory does not cover the presently observed resonance range. It may be that CF vortex number transitions, if present in these samples, are strongly affected by the disorder, so that the ν^* where it occurs is different than predicted.

One of the most striking features of the data are the peaks around $\nu = 0.72$ to 0.74 , not far from $\nu = 3/4$ where $2p = 4$ CFs form a Fermi surface, and move in a zero effective magnetic field. It may be that this region is characterized by a CF solid more like that considered for $B = 0^{45}$, in which the particle size is determined by competition between kinetic energy and interaction; such a solid would need to be stabilized by the dense homogenous disorder.

One possible cause of the large damping would be pockets of FQH liquid within the insulating phase. It is unlikely the pockets are localized in the Al disorder, though it may be that the much weaker, large length scale disorder associated with the remote ionized donors is containing liquid pockets, as described in ref. 46. An inhomogeneous microemulsion phase of the sort proposed in refs. 47,48 is also conceivable. If there are pockets of CF liquid, the additional damping for $\nu \sim 3/4$ is explainable as due to dissipation in the liquid pockets. The increase of f_{pk} near that frequency could involve the pockets to be screening the Coulomb interaction between carriers of the solid, reducing its effective stiffness at long range.

In summary we have found that Al-alloy disordered samples exhibit the signature of an IQHEWS for $|\nu^*|$ below about 0.2. The IQHEWS is remarkably similar to that in ultralow disorder samples, and shows some likely effects of skyrmions. For $\nu < 0.8$, where resonances are not seen for ultralow disorder samples, the resonance has an evolution that is more complex than that seen in low disorder samples. f_{pk} increases with decreasing ν as ν goes below 0.8, and a maximum in f_{pk} vs ν occurs for ν just below $3/4$. The CF picture provides possible explanations of the features. The disorder extends the ν range of the existence of the resonance, so that there is reentrance at least of a highly damped resonance, around FQHE states.

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